# Math 2200 Chapter 1 Arithmetic and Geometric Sequences and Series Review

<table>
<thead>
<tr>
<th>Key Ideas</th>
<th>Description or Example</th>
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<tr>
<td><strong>Sequences</strong></td>
<td>An ordered list of numbers where a mathematical pattern can be used to determine the next terms. Example: 1, 5, 9, 13, 17... or 1000, 100, 10, 1... $n$ is the term position or the number of terms, $n$ must be a natural number.</td>
</tr>
<tr>
<td><strong>Series</strong></td>
<td>The sum of all the terms of a finite sequence. Example: 5 + 10 + 15 + 20 $\frac{1}{2}$ + $\frac{1}{4}$ + $\frac{1}{8}$...</td>
</tr>
<tr>
<td><strong>Arithmetic Sequence</strong></td>
<td>A sequence that has a common difference, $d = t_n - t_{n-1}$ Example: 2, 4, 6, 8, 10, 12, 14... where $d = 2$</td>
</tr>
<tr>
<td><strong>Graph of an Arithmetic Sequence</strong></td>
<td>Always discrete since the $n$ values or the term position must be natural numbers. Related to a linear function $y = mx + b$ where $m = d$ and $b = t_1 - m$. The slope of the graph represents the common difference of the general term of the sequence. $t_1 = b + m$, add the $y$-intercept to the slope to get the value of the first term of the sequence.</td>
</tr>
<tr>
<td><strong>Arithmetic Series</strong></td>
<td>The sum of an arithmetic sequence. Use $S_n = \frac{n}{2}(t_1 + t_n)$ when you know the first term, last term, and the number of terms. Use $S_n = \frac{n}{2}[2t_1 + (n - 1)d]$ when you know the first term, the common difference, and the number of terms. You may need to determine the number of terms by using $t_n = t_1 + (n - 1)d$.</td>
</tr>
<tr>
<td><strong>Geometric Sequence</strong></td>
<td>A sequence that has a common ratio, $t_n = t_1 r^{n-1}$ Example: 3, 9, 27, 82, 243, 729, 2187... where $r = 3$</td>
</tr>
</tbody>
</table>

Graph is discrete, not continuous, and not linear.
### Problem

A ball is dropped from a height of 100 m. After each bounce it rises to 40% of its previous height.

a) Write the first term and the common ratio of the geometric sequence.

\[ t_1 = 100 \quad r = 0.4 \]

b) Write the general term of the sequence that relates the height of the bounce to the number of bounces.

\[ t_n = 100(0.4)^{n-1} \]

c) What height does the ball reach after the 8th bounce?

\[ n = 9 \quad t_9 = 100(0.4)^8 \quad t_9 = 0.066 \]

0.066 m = 6.6 cm

### Finite Geometric Series

A finite geometric series is the expression for the sum of the terms of a finite geometric sequence.

The **General formula** for the Sum of the first \( n \) terms

\[
S_n = \frac{t_1(r^n - 1)}{r - 1}, \quad r \neq 1
\]

Known Values are: \( t_1, r \) and \( n \)

\[
S_n = \frac{rt_n - t_1}{r - 1}, \quad r \neq 1
\]

Known Values are: \( t_1, r \) and \( t_n \)

### Infinite Geometric Series

A geometric series that does not end or have a final term.

It may be **convergent** (sum approaches a value, there is a formula for this) or **divergent** (sum gets infinitely larger).

The series is convergent if the absolute value of \( r \) is a fraction or decimal less than one:

\[ |r| < 1, \quad -1 < r < 1 \]

The series is divergent if the absolute value of \( r \) is greater than one:

\[ |r| > 1, \quad -1 > r > 1 \]

### Sum of an Infinite Geometric Series, only if it converges

You must know the value of the first term and the common ratio.

\[
S_\infty = \frac{t_1}{1-r} |r| < 1
\]

### General or explicit formula

- The unique parameters are substituted into the formula.
- The parameters are \( t_1 \) and \( r \) for a geometric sequence or \( d \) for arithmetic sequence.
- Example: \( t_n = 3 \cdot 2^{n-1} \) or \( t_n = 2n + 3 \)
**Math 2200 Chapter 2 Trigonometry Review**

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<tr>
<td>Sketching an angle $\theta$ in standard position. The measure of an angle in standard position can be between $0^\circ$ to $360^\circ$. The symbol $\theta$ is often used to represent the measure of an angle.</td>
<td>The vertex of the angle is located at the origin (0,0) on a Cartesian plane. The initial arm of the angle lies along the positive x-axis. The terminal arm rotates in a positive direction. The angle in standard position is measured from the positive x-axis.</td>
</tr>
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| Each angle in standard position has a corresponding reference angle. Reference angles are positive acute angles (< $90^\circ$) measured from the terminal arm to the nearest x-axis. Any angle from $90^\circ$ to $360^\circ$ is the reflection in the x-axis and/or the y-axis of its reference angle. | ![Reflection of Reference Angles](reflection.png) |

| Consider a reference right angle triangle The three primary trigonometry ratios are $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$. To calculate a ratio given an angle using your calculator: $\sin 30^\circ = \frac{1}{2}$ or 0.5 The mode of your calculator should be in degrees. | ![Reference Right Angle Triangle](triangle.png) |

| Calculator: Determine Approximate Trig Ratios (four decimal places) 1. $\sin 25^\circ = 0.4226$ 2. $\cos 121^\circ = -0.5150$ 3. $\tan 335^\circ = -0.4663$ 4. $\sin 0^\circ = 0$ 5. $\tan 90^\circ = \text{undefined}$ The ratios are given as decimal approximations. | ![Calculator](calculator.png) |
Given any point P(x, y) on the Terminal Arm of an angle in standard position, the Pythagorean Theorem can be used to determine the distance the point is from the origin.

This distance can be labeled r.

The x- and y- coordinates of the point can be used to determine the exact values for the primary trig ratios.

The points P(x, y), P(−x, y), P(−x, −y) and P(x, −y) are points on the terminal sides of angles in standard position that have the same reference angle. These points are reflections of the point P(x, y) in the x-axis, y-axis or both axes.

The trigonometry ratios may be positive or negative in value depending on which quadrant the terminal arm is in. A point on the terminal arm would have coordinates (x, y).

“A Smart Trig Class”

There are two special right triangles for which you can determine the exact values of the primary trigonometric ratios.

TIP: the smallest angle is always opposite the shortest side.

Determining the measure of an angle, given the defining ratio.

Use the inverse trig ratios.

Determine the Measure of the Angle Given a Trig Ratio

Determine the measure of angle A, to the nearest degree:

\[ \theta = \frac{3}{5} \]

\[ \theta = 360° \]

Enter a positive ratio in your calculator to produce the reference angle,

\[ \sin^{-1}(0.6) \]

<table>
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<tr>
<th>Quadrants</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 50.5632</td>
<td>34°</td>
<td>146°</td>
<td>104°</td>
<td>256°</td>
</tr>
<tr>
<td>A = 0.7542</td>
<td>41°</td>
<td>121°</td>
<td>219°</td>
<td>329°</td>
</tr>
<tr>
<td>A = 1.5643</td>
<td>58°</td>
<td>128°</td>
<td>208°</td>
<td>338°</td>
</tr>
</tbody>
</table>

Quadrantal Angles

If the terminal arm lies on an axis, the angle is called a Quadrantal angle (it separates the quadrants).

The quadrants are labeled in a counterclockwise direction.

The Quadrantal angles for one revolution are 0°, 90°, 180°, 270°, 360°
Ratios of Quadrantal Angles

To determine, without the use of technology, the value of \( \sin \theta \), \( \cos \theta \) or \( \tan \theta \), given any point \( P (x, y) \) on the terminal arm of angle \( \theta \), where \( \theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ \) or \( 360^\circ \) use the defining ratios and consider any point on the axis. Plot the point, determine the values of \( x \), \( y \), and \( r \) and calculate the ratio.

The Sine Law
An angle and its opposite side create a defining ratio that can be used to calculate the other measurements.

Notice you are given the measurements of an angle and its opposite side.

To determine side \( c \)
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
To determine \( c \)
\[
\frac{20}{\sin 31^\circ} = \frac{c}{\sin 104^\circ}
\]
\( c = \frac{20 \sin 104^\circ}{\sin 31^\circ} \approx 37.7 \)

The Ambiguous Case of the Sine Law
Used when you are finding an Angle.

Given: two angles and one side or two sides and an angle opposite one of the given sides

An angle and its opposite side are given to define one ratio \( \frac{a}{\sin A} \) of the Sine Law.

Since angle B could be obtuse or acute, the ambiguous case must be considered.

**Angle A is Obtuse**

You must compare the length of side \( a \) to side \( b \).

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<tr>
<th>Number of Triangles</th>
<th>Sketch</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td><img src="image1.png" alt="Sketch" /></td>
<td>( a \leq b )</td>
</tr>
<tr>
<td>1</td>
<td><img src="image2.png" alt="Sketch" /></td>
<td>( a &gt; b )</td>
</tr>
</tbody>
</table>

**Angle A is Acute**

You must compare the length of side \( a \) to side \( b \) and also the height of the triangle, \( h \).

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<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td><img src="image3.png" alt="Sketch" /></td>
<td>( a &lt; h )</td>
</tr>
<tr>
<td>1</td>
<td><img src="image4.png" alt="Sketch" /></td>
<td>( a = h )</td>
</tr>
<tr>
<td>2</td>
<td><img src="image5.png" alt="Sketch" /></td>
<td>( a = b \sin A )</td>
</tr>
<tr>
<td>3</td>
<td><img src="image6.png" alt="Sketch" /></td>
<td>( h \leq a &lt; b )</td>
</tr>
<tr>
<td>4</td>
<td><img src="image7.png" alt="Sketch" /></td>
<td>( b \sin A &lt; a &lt; b )</td>
</tr>
</tbody>
</table>

The Cosine Law
Describes the relationship between the cosine of an angle and the lengths of the three sides of any triangle.

\[ a^2 = b^2 + c^2 - 2bc \cos A \quad \text{or} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc} \]
Math 2200 Chapter 3 Quadratic Functions Review

<table>
<thead>
<tr>
<th>Key Ideas</th>
<th>Description or Example</th>
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<tbody>
<tr>
<td>Quadratic Functions</td>
<td>polynomial of degree two</td>
</tr>
<tr>
<td>For a quadratic function, the graph is in the shape of a parabola</td>
<td>Parent Graph is $y = x^2$</td>
</tr>
<tr>
<td>Characteristics of a Quadratic Function from the vertex form of the equation $f(x) = a(x - p)^2 + q$</td>
<td>- The coordinates of the vertex are at $(p, q)$. Note that the negative symbol from $(x - p)$ does not transfer to the value of $p$. $f(x) = 2(x - 3)^2 + 4$ vertex at $(3, 4)$; $f(x) = 2(x + 3)^2 + 4$ vertex at $(-3, 4)$</td>
</tr>
<tr>
<td>Horizontal Translations: When $p &gt; 0$ the graph moves (translated) to the right. When $p &lt; 0$ is translated or shifts to the left. Vertical Translations: When $q &gt; 0$ the parabola shifts up. When $q &lt; 0$ the parabola shifts down.</td>
<td></td>
</tr>
<tr>
<td>- The parameter $a$ indicates the direction of opening as well as how narrow or wide the graph is in relation to the parent graph. When $a &gt; 0$, the graph opens up and the vertex is a minimum. When $a &lt; 0$, the graph opens downward and the vertex is a maximum. When $-1 &lt; a &lt; 1$, the parabola is wider than the parent graph. When $a &gt; 1$ or $a &lt; -1$, the parabola is narrower than the parent graph.</td>
<td></td>
</tr>
<tr>
<td>- The Axis of Symmetry is an imaginary vertical line through the vertex that divides the function graph into two symmetrical parts. The equation for the axis of symmetry is represented by $x = p$. The only exception is for real life models.</td>
<td></td>
</tr>
<tr>
<td>- The range of a quadratic function depends on the value of the parameters $&quot;a&quot;$ and $&quot;q&quot;$. When $&quot;a&quot;$ is positive, the range is ${y \mid y \geq q}$. When $&quot;a&quot;$ is negative, the range is ${y \mid y \leq q}$.</td>
<td></td>
</tr>
</tbody>
</table>
| Summary of Characteristics. **Determine the Number of $x$-Intercepts Using a and $q$.** | ]
| $f(x) = a(x - p)^2 + q$ |
| $x$- and $y$-intercepts | Calculate the $x$-intercept $pt (x, 0)$ by replacing $y$ with 0 in either form of the function equation. $ax^2 + bx + c = 0$ or $a(x - p)^2 + q = 0$. Calculate the $y$-intercept $(0, y)$ by replacing $x$ with 0 and solving for the value of $y$. |
| Write a quadratic function in the form $y = a(x - p)^2 + q$ for a given graph or a set of characteristics of a graph. | Write the equation of the Quadratic Function in the form $f(x) = a(x - p)^2 + q$. |
Converting from vertex form to standard form. Follow order of operations PEMDAS

\[ y = 2(x + 3)^2 + 1 \]
\[ y = 2(x + 3)(x + 3) + 1 \]
\[ y = 2(x^2 + 6x + 9) + 1 \]
\[ y = 2x^2 + 12x + 18 + 1 \]

Completing the square Converting the standard form into vertex form so the characteristics can be determined.

\[ y = a(x - p)^2 + q \]

Complete the Square Process
The middle term coefficient must be divided by two and then squared.

Complete the Square when the leading coefficient is not 1.
The parameter “a” must be factored out of the terms involving x before completing the square.

Solving Problems

Solving Problems Revenue

Problem 5 Revenue
How many pear trees should be planted to maximize the revenue from an orchard for one year? Research for an orchard has shown that, if 100 pear trees are planted, then the annual revenue is $90 per tree. The annual revenue per tree is reduced by $0.70 for every additional tree planted.

\[ \text{Revenue} = (2)(90) \]
\[ \text{Revenue} = (90)(100) \]
\[ \text{Maximum Revenue} = (90 - 0.7x)(100 + 1x) \]

Vertex (14,28, 9142,86)

14 trees should be planted to maximize the revenue
## Math 2200 Chapter 4 Quadratic Equations Review

<table>
<thead>
<tr>
<th>Key Ideas</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quadratic Expression</strong></td>
<td>( ax^2 + bx + c ) [ a(x-p)^2 + q ]</td>
</tr>
<tr>
<td><strong>Factoring Quadratic Expressions</strong></td>
<td>( ax^2 + bx + c ) [ a(f(x))^2 + bf(x) + c ] [ a(x-p)^2 + q ] [ a^2x^2 - b^2y^2 ] [ a^2(f(x))^2 - b^2(g(y))^2 ]</td>
</tr>
<tr>
<td><strong>Quadratic Equation</strong></td>
<td><strong>Description:</strong> an equation in which the degree of the polynomial expression is two. ( ax^2 + bx + c = 0 ) [ ax^2 + bx = c ]</td>
</tr>
<tr>
<td><strong>Solve by Graphing</strong></td>
<td>Graph the related quadratic function and determine the ( x )-intercepts of the graph. When using graphing technology, navigate to the menu containing zeros (where the height of the graph of the function is zero). Solve ( x^2 - 5x + 6 = 0 ). Graph the corresponding function ( f(x) = x^2 - 5x + 6 )</td>
</tr>
<tr>
<td><strong>Zero Product Property:</strong></td>
<td>If the factors of a quadratic equation have a product of zero, then one or both of the factors must be equal to zero. ( (x+3)(x-9) = 0 ) [ x + 3 = 0 \text{ or } x - 9 = 0 ] [ x = -3 \text{ or } x = 9 ]</td>
</tr>
<tr>
<td><strong>Solve by Factoring</strong></td>
<td>( 3x^2 + 19x - 14 = 0 ) [ (3x - 2)(x + 7) = 0 ] [ 3x - 2 = 0 \text{ or } x + 7 = 0 ] [ x = \frac{2}{3} \text{ or } x = -7 ] [ x^2 - 36 = 0 ] [ x^2 = 36 ] [ x = 6 \text{ or } x = -6 ] [ x = \pm 6 ] [ 6x^2 + x - 15 = 0 ] [ 6x^2 + 10x - 9x - 15 = 0 ] [ 2x(3x + 5) - 3(3x + 5) = 0 ] [ (3x + 5)(2x - 3) = 0 ] [ (3x + 5) = 0 \text{ or } (2x - 3) = 0 ] [ x = -5/3 \text{ or } x = 3/2 ]</td>
</tr>
<tr>
<td><strong>Solving a Difference of Squares</strong></td>
<td>[ 16x^2 - 121 = 0 ] [ (4x)^2 - (11)^2 = 0 ] [ (4x - 11)(4x + 11) = 0 ] [ (4x - 11) = 0 \text{ or } (4x + 11) = 0 ] [ x = \pm 11/4 ] [ 5(x+1)^2 - 80 = 0 ] [ 5[(x+1)^2 - 16] = 0 ] [ (x+1)^2 - 16 = 0 ] [ <a href="%5Bx+1%5D+4">(x+1)^2-4</a> = 0 ] [ [x = -3] [(x+5) = 0 ] [ x = 3 \text{ or } x = -5 ]</td>
</tr>
<tr>
<td><strong>Completing the square</strong></td>
<td><strong>Description:</strong> The process of rewriting a quadratic polynomial from the standard form</td>
</tr>
</tbody>
</table>

**The Roots of a quadratic equation are the solutions to the quadratic equation.** The roots of the equation are related to the **zeros** of the related quadratic function. The roots of the equation are related to the **\( x \)-intercepts** of the graph of the related quadratic function.

\[ \text{Roots of equation} \]
\[ \text{Zeros of the function} \]
\[ \text{\( x \)-intercepts of graph} \]
Take the square root of both sides of the equation to solve for the variable. Remember both the positive and negative root could be solutions.

Quadratic formula can be used to determine the roots of an equation of the quadratic are not easily factorable or if the roots are irrational.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Description: you can use this formula to solve a quadratic equation of the form, \( ax^2 + bx + c = 0 \), \( a \neq 0 \)

Example: \( 2x^2 - 12x - 4 = 0 \)

\[ x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(-4)}}{2(2)} \]
\[ x \approx 6.32 \text{ or } -0.32 \]

Use the Discriminant to determine the nature of the roots.

The express under the radical symbol (radicand) in the quadratic formula: \( b^2 - 4ac \)

Three cases:
- \( b^2 - 4ac > 0 \) two distinct real roots
- \( b^2 - 4ac = 0 \) one distinct root or two equal real roots
- \( b^2 - 4ac < 0 \) no real roots

**Problem Solving**

Find the lengths of the two unknown sides of a triangle if the hypotenuse is 15 cm long and the sum of other two legs is 21 cm.

Let \( x \) be one side of the triangle, \( 21 - x \) will be the other side.

\[ x^2 + (21 - x)^2 = 15^2 \]

\[ x^2 + 441 - 42x + x^2 = 225 \]

\[ 2x^2 - 42x + 226 = 0 \]

Find the dimensions of the rectangle where the length of a rectangle is 4 inches more than its width. The area of the rectangle is 91 square inches.

Step 1: Given the picture of the rectangle.

Step 2: Write the equation using the formula Area = length \( \times \) width

Step 3: Solve the equation.

The length is 10 in and the width is 12 in.

Find two consecutive odd integers where their product is less than fourteen times their sum. Two consecutive odd integers can be written in the form \( n \) and \( n + 2 \).

Step 1: Let \( x = \) the first integer.

Step 2: Write the equation.

Step 3: Solve the equation.

There are two solutions of integers.

Find two consecutive even integers where their product is less than fourteen times their sum. Two consecutive even integers can be written in the form \( n \) and \( n + 2 \).

Step 1: Let \( x = \) the first integer.

Step 2: Write the equation.

Step 3: Solve the equation.

There are two solutions of integers.

The hypotenuse of a right triangle is 4 more than the shorter leg. The longer leg is 3 more than the shorter leg. Find the length of the smaller leg.

The hypotenuse is 5 more than the longer leg.

Step 1: Write the equation

Step 2: Solve the equation.
### Key Ideas

**Radical means root. The index determines which root you are looking for.**

**Description or Example**

- **Principle Square Root**
  - \( \sqrt{16} = +4 \) the root is positive
  - \( \sqrt{16} = -4 \) the radical is negative, the root will be negative

- **Negative Square Root**

- **Equations**
  - Even degrees have \( \pm \pm \) roots.
  - Odd degrees have one root.

- **Perfect Squares**
  - \( 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169 \ldots \)
  - **Perfect Cubes**
  - \( 1, 27, 64, 125 \ldots \)
  - **Perfect Fourth Roots**
  - \( 1, 16, 81 \ldots \)

- **Convert entire radicals to mixed radicals**

- **Convert mixed radicals to entire radicals**

- **Comparing and ordering radical expressions**

- **Identifying restrictions on the values for a variable in a radical expression**
  - The radicand must be greater or equal to zero.
  - For \( \sqrt{x} \) the restriction is \( x \geq 0 \), \( x \in \mathbb{R} \)
  - For \( \sqrt{2x - 5} \) the restriction is \( x \geq \frac{5}{2}, x \in \mathbb{R} \)

- **Simplifying radical expressions using addition or subtraction**

- **Multiply radicals**

- **Divide radicals**

### Table

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<td>![Radical Symbol]</td>
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<td><strong>Principal Square Root</strong></td>
<td>( \sqrt{16} = +4 ) the root is positive</td>
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<td><strong>Negative Square Root</strong></td>
<td>( \sqrt{16} = -4 ) the radical is negative, the root will be negative</td>
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<td><strong>Equations</strong></td>
<td>Even degrees have ( \pm \pm ) roots. Odd degrees have one root.</td>
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<td><strong>Perfect Squares</strong></td>
<td>( 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169 \ldots )</td>
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<tr>
<td><strong>Convert entire radicals to mixed radicals</strong></td>
<td>( \sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} )</td>
</tr>
<tr>
<td><strong>Convert mixed radicals to entire radicals</strong></td>
<td>( \sqrt{16} = \sqrt{8 \times 2} = 2\sqrt{2} )</td>
</tr>
<tr>
<td><strong>Comparing and ordering radical expressions</strong></td>
<td>Write as entire radicals to compare, apply the proper index. Write as a decimal and compare.</td>
</tr>
<tr>
<td><strong>Identifying restrictions on the values for a variable in a radical expression</strong></td>
<td>For ( \sqrt{x} ) the restriction is ( x \geq 0 ), ( x \in \mathbb{R} ) For ( \sqrt{2x - 5} ) the restriction is ( x \geq \frac{5}{2}, x \in \mathbb{R} )</td>
</tr>
<tr>
<td><strong>Simplifying radical expressions using addition or subtraction</strong></td>
<td>Radicals must have the <strong>same index</strong> and the <strong>same radicand</strong> to be added or subtracted. Only add or subtract the coefficients. Do not add radicands.</td>
</tr>
<tr>
<td><strong>Multiply radicals</strong></td>
<td>Radicals must have the <strong>same index</strong>. Multiply coefficients, multiply radicands, simplify.</td>
</tr>
<tr>
<td><strong>Divide radicals</strong></td>
<td>Radicals must have the <strong>same index</strong>. Divide coefficients, divide radicands, simplify.</td>
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</table>
### Rationalize the Denominator

**Monomial Denominator**

\[
\frac{1}{\sqrt{3}} \times \frac{\sqrt{5}}{5} = \frac{\sqrt{5}}{10}
\]

\[
\sin 45^\circ = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}
\]

### Binomial Denominator

\[
\frac{5}{6 + \sqrt{3}} \times \frac{6 - \sqrt{3}}{6 - \sqrt{3}} = \frac{30 - 6\sqrt{3}}{3} = 10 - 2\sqrt{3}
\]

### Solve an Equation with One Radical Term

State restrictions on the variable in the radicand. Check for extraneous roots.

\[
\sqrt{x + 3} = 4
\]

**Radicand Restrictions**

\[
x + 3 \geq 0
\]

\[
x \geq -3
\]

Verify by substitution:

\[
\sqrt{x + 3} = 4
\]

\[
(\sqrt{x + 3})^2 = 4^2
\]

\[
x + 3 = 16
\]

\[
x = 13
\]

\[
\sqrt{x + 2} = 4 - x
\]

\[
(\sqrt{x + 2})^2 = (4 - x)^2
\]

\[
x + 2 = 16 - 8x + x^2
\]

\[
0 = 14 - 9x + x^2
\]

\[
0 = x^2 - 9x + 14
\]

\[
0 = (x - 7)(x - 2)
\]

\[
x = 7, 2
\]

Therefore, the solution is \(x = 2\).

### Solve an Equation with Two Radical Terms

State restrictions on the variable in the radicand. Check for extraneous roots.

\[
4\sqrt{x} = \sqrt{x + 60}
\]

**Restrictions** \(x \geq 0\), \(x \neq -60\)

\[
(4\sqrt{x})^2 = (\sqrt{x + 60})^2
\]

\[
4^2 = \sqrt{x + 60}
\]

\[
16x = x + 60
\]

\[
15x = 60
\]

\[
x = 4
\]
### Key Ideas

#### Simplifying Rational Expressions

A rational expression is a fraction, \( \frac{p}{q} \), where \( p \) and \( q \) are polynomials, \( q \neq 0 \).

A non-permissible value is a value of the variable that causes an expression to be undefined. For a rational expression, this occurs when the denominator is zero. Indicate all non-permissible values for variables in a rational expression.

Rational expressions can be simplified by:
- factoring the numerator and the denominator
- determining non-permissible values for variables
- divide all common factors in both the numerator and denominator

**Example:**

\[
\frac{4x^2 - 9}{2x^2 - x - 3} = \frac{(2x - 3)(2x + 3)}{(2x + 3)(x + 1)}
\]

Non-permissible values (NPVs): \( x = \frac{3}{2} \) and \( x = -1 \)

\[
= \frac{2x - 3}{x + 1}, \quad x \neq \frac{3}{2}, -1
\]

#### Adding and Subtracting Rational Expressions

To add or subtract rational expressions, the expressions must have the same denominator.

As with fractions, we add or subtract rational expressions with the same denominator by combining the terms in the numerator and then writing the result over the common denominator.

For terms with **Like Denominators**, add or subtract the numerators only. The denominator does not change.

For **unlike denominators**, rewrite them in equivalent forms that have the same denominator.

- Factor each denominator.
- Find the least common denominator. The LCD is the product of all different factors from each denominator, with each factor raised to the greatest power that occurs in any denominator.

**Example:**

\[
\frac{x + 1}{x^2 + 4x + 4} - \frac{6}{x^2 - 4} = \frac{x + 1}{(x + 2)(x + 2)} - \frac{6}{(x - 2)(x + 2)}
\]

\[
= \frac{(x + 1)(x - 2)}{(x + 2)(x + 2)(x - 2)} - 6\frac{(x + 2)}{(x + 2)(x - 2)(x + 2)}
\]

\[
= \frac{(x + 1)(x - 2)(x - 2) - 6(x + 2)(x - 2)}{(x + 2)^2(x - 2)}
\]

\[
= \frac{x^2 - 7x - 14}{(x + 2)^2(x - 2)}, \quad x \neq -2
\]

**LCD is:** \( (x + 2)(x - 2) \)
### Multiplying Rational Expressions

**To Multiply Rational Expressions:** (a common denominator is not required)

- Factor the polynomials in each numerator and denominator
- Simplify the expression by dividing out common factors in both the numerator and denominator. *Don’t forget to simplify before you multiply!*
- State the non-permissible values

\[
\frac{7x+7}{x+4} \cdot \frac{x^2-x-20}{7x^2-42x-49}
\]

\[
= \frac{7(x+1)(x-5)}{7(x+7)(x+1)} \cdot \frac{(x-5)(x+4)}{7(x-7)(x+1)}
\]

\[
= \frac{4x-5}{x-7} \cdot x = -1, -4, 7
\]

### Dividing Rational Expressions

**To Divide Rational Expressions:**

- Factor the polynomials in the numerators and denominators if possible
- List all non-permissible values for the variables.
- Multiply the first term by the reciprocal of the second term
- Divide out common factors

\[
\frac{x^2+3x-10}{x^2+6x+5} \div \frac{2x^2-x-3}{2x^2+x-6} = \frac{8x+20}{6x+15}
\]

\[
= \frac{(x-5)(x+2)}{(x+5)(x-2)(x+2)(x+4)(x+5)}
\]

\[
= \frac{3(x-2)}{4(x+2)} \cdot x = -\frac{3}{2}, -5, -2, -1
\]

### Solving Rational Equations

**To Solve a rational equation:**

- Determine the LCD of the denominators, list all NPV’s
- Multiply both sides of the equation by the LCD. Reduce common factors.
- Solve the resulting polynomial equation.
- Verify all solutions

#### Solve Rational Equations

a) \[ \frac{7}{x+2} = \frac{6}{x-5} \]

**Domain:** \( \{x | x \neq -2, x \neq 5, x \in \mathbb{R} \} \)

\[
\frac{1}{(x+2)(x-5)} \cdot \frac{6}{1} = \frac{7}{x+2}
\]

\[
7(x-5) = 6(x+2)
\]

\[
x = 47
\]
Solving Problems

1. A traveling salesman drives from home to a client’s store 150 miles away. On the return trip he drives 10 miles per hour slower and adds one-half hour in driving time. At what speed was the salesman driving the way to the client’s store?

Let \( r \) be the rate of travel (speed) in miles per hour.

<table>
<thead>
<tr>
<th>Distance (miles)</th>
<th>Rate (miles/hour)</th>
<th>Time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip to client</td>
<td>150</td>
<td>( \frac{180}{r} )</td>
</tr>
<tr>
<td>Trip home</td>
<td>150</td>
<td>( \frac{150}{r-10} )</td>
</tr>
</tbody>
</table>

**Longer Time (slower speed) - Shorter Time (faster speed) = Time Difference**

\[
\frac{150}{r-10} - \frac{150}{r} = \frac{1}{2} \text{ hour}
\]

\[
LCM = 2r(r-10)
\]

\[
\frac{150(r-10)}{r-10} - \frac{150r}{r-10} = \frac{1}{2} \cdot \frac{2(r-10)}{r-10}
\]

\[
300r - 300(r-10) = r(r-10)
\]

\[
300r - 300r + 3000 = r^2 - 10r
\]

\[
0 = r^2 - 10r - 3000
\]

\[
0 = (r-60)(r+50)
\]

\[
r = 60 \text{ or } -50
\]

Why is 50 not an acceptable answer?

The salesman drove from home to the client’s store at 60 miles per hour.

Solving Problems

2. If a painter can paint a room in 4 hours and her assistant can paint the room in 6 hours, how many hours will it take them together?

Let \( t \) be the time it takes them to paint the room together.

<table>
<thead>
<tr>
<th>Rate of Work</th>
<th>Time Worked</th>
<th>Fraction of Work Done by Painter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Painter</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{t}{4} )</td>
</tr>
<tr>
<td>Assistant</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{t}{6} )</td>
</tr>
</tbody>
</table>

Write an equation, in terms of \( t \), to represent the time it takes to paint the room working together.

\[
\left( \frac{t}{4} \right) + \left( \frac{t}{6} \right) = \left( \frac{1}{6} \right)
\]

Working together they will paint the room in 2.4 hours.

**Examples**

<table>
<thead>
<tr>
<th>Name of Factoring Method</th>
<th>Characteristics</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greatest Common Factor</td>
<td>Each term of the polynomial has a common factor</td>
<td>( 3x^2 + 9xy - 12 = 3(x^2 + 3xy - 4) )</td>
</tr>
<tr>
<td>Sum and Product</td>
<td>Trinomial with even degree where the coefficient of the highest degree term is 1</td>
<td>( x^2 + 2x + 15 = (x + 3)(x + 5) )</td>
</tr>
<tr>
<td>Grouping</td>
<td>Four term polynomal where each group of two terms has a common factor</td>
<td>( y^2 - y^2 = 0 )</td>
</tr>
<tr>
<td>Difference of Two Squares</td>
<td>Binomial with two perfect square terms written as a difference</td>
<td>( a^2 - b^2 = (a + b)(a - b) )</td>
</tr>
<tr>
<td>Perfect Square Trinomial</td>
<td>Trinomial with perfect square terms for the first and last terms, and the middle term is twice the product of the square roots of the first and last terms</td>
<td>( 16y^2 - 20y + 64 = (4y - 8)(4y + 8) )</td>
</tr>
<tr>
<td>General Trinomial</td>
<td>Trinomial must be factorable with rational coefficients and constants that cannot be factored by one of the previous methods</td>
<td>( 6x^2 + 12x + 12 = 2(x + 3)(3x + 2) )</td>
</tr>
</tbody>
</table>

**Common Errors**

When simplifying rational expressions, an error is to divide only one term in the dividend by the divisor.

<table>
<thead>
<tr>
<th>Incorrect</th>
<th>Correct</th>
</tr>
</thead>
</table>
| \[
\frac{3x + 6}{3x} = \frac{2x + 6}{2x}
\] | \[
\frac{3x + 6}{3x} = \frac{3(x + 2)}{3x} = \frac{x + 2}{x}
\] |
**Math 2200 Chapter 7 Absolute Value and Reciprocal Functions Review**

### Key Ideas

<table>
<thead>
<tr>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Absolute value</strong></td>
</tr>
<tr>
<td>Represents the distance from zero on a number line, regardless of direction. Absolute value is written with a vertical bar around a number or expression. It represents a positive value.</td>
</tr>
<tr>
<td>Example: $</td>
</tr>
<tr>
<td>$</td>
</tr>
<tr>
<td>The absolute value of a positive number is positive. The absolute value of a negative number is positive, and the absolute value of zero is zero.</td>
</tr>
</tbody>
</table>

**Graphing an Absolute Value Function**

**$y = |2x + 3|$**

To graph the absolute value of a linear function:

- **Method 1:** Create a table of values, then graph the function using the points. Note: There are two pieces and all points are on or above the x-axis.

- **Method 2:** Graph the related linear function $y = 2x + 3$. Reflect, in the x-axis, the part of the graph that is below the x-axis. (Negative $y$-values become positive.)

  - The domain is all real numbers.
  - The range is $\{y \mid y \geq 0\}$.

**$y = |x^2 - 4|$**

To graph the absolute value of a quadratic function:

- **Method 1:** Create a table of values, and then graph the function using the points. Note there are three pieces and all points are on or above the x-axis.

- **Method 2:** Graph the related quadratic function $y = x^2 - 4$. Then reflect in the x-axis the part of the graph that is below the x-axis.

  - The domain is all real numbers.
  - The range is $\{y \mid y \geq 0\}$.

### Writing Absolute Value as a Piecewise Function

1. Determine the $x$-intercepts by setting the expression within the absolute value equal to zero.
2. Use slope (linear function) or direction of opening (quadratic function) to determine which parts of the graph are above or below the x-axis.
3. Keep the parts that are positive (above x-axis) and indicate the domain.
4. Reflect the negative parts in the x-axis, multiply the expression by -1 for this part and indicate the domain.

**Express as a piecewise function.** $y = |2x - 4|

- **Invariant point:** $x$-intercept $2x - 4 = 0$  
  $x = 2$

  ![Graph of $y = |2x - 4|$](image)

- As a piecewise function $y = |2x - 4|$ would be $|2x - 4| = \begin{cases} 2x - 4 & \text{if } x \geq 2 \\ -(2x - 4) & \text{if } x < 2 \end{cases}$

  Be careful when assigning domain, it changes depending on which piece of the graph was below the x-axis. Note the linear expression would have a negative slope, examine how this changes the domain pieces.

**Linear Expressions**

$|2x + 3| = \begin{cases} 2x + 3 & \text{if } x \geq \frac{3}{2} \\ -(2x + 3) & \text{if } x < \frac{3}{2} \end{cases}$

**Quadratic Expressions**

$|x^2 - 3x - 4| = \begin{cases} x^2 - 3x - 4 & \text{if } -1 \geq x \geq 4 \\ -(x^2 - 3x - 4) & \text{if } -1 < x < 4 \end{cases}$
Analyzing Absolute Value Functions Graphically

To analyze an absolute value graphically:
- first, graph the function.
- then, identify the characteristics of the graph, such as x-intercept, y-intercept, minimum values, domain and range.

The domain of an absolute value function, \( y = |f(x)| \), is the same as the domain of \( y = f(x) \).
The range of the absolute value function will be greater or equal to zero.

Solving an Absolute Value Equation Graphically

An absolute value equation includes the absolute value of an expression involving a variable. To solve graphically:
- Graph the left side and the right side of the equation on the same set of axes.
- The point(s) of intersection are the solutions.

Solving an Absolute Value Equation Algebraically

To solve algebraically consider the two cases:
- Case 1: the expression inside the absolute value symbol is greater than or equal to zero.
- Case 2: the expression in the absolute value symbol is less than zero.

The roots in each case are the solutions.

There may be extraneous roots that need to be identified and rejected.
Verify the solution by substituting into the original equation.
There may be no solution, one solution or two solutions if the absolute value expression is a line.
There may be no solution, one, two or three solutions if the absolute value expression is quadratic.

Solving Linear Abs Equation

\[ |x + 2| = 5 \]

Determine the zeros of the abs function. Determine domain pieces.

\(-x + 2 = 0\)
\(x = 2\)

\[ |−x + 2| = \begin{cases} −x + 2, & x \leq 2 \\ −(−x + 2), & x > 2 \end{cases} \]

Case 1: positive y values
\(-x + 2 = 5\)
\(-x = 3\)
\(x = -3\)

Case 2: negative y values
\(x \geq 2\)
\(-(-x + 2) = 5\)
\(-x - 2 = 5\)
\(x = -7\)

Verify each solution in the original absolute value equation.

\(|x + 3| = -2\) does not have any solution, absolute value must be positive.

Solving Quadratic Abs Equations

\[ x^2 + 5x + 4 = 10 \]

Determine the zeros of the abs function.
\(x^2 + 5x + 4 = 0\)

\[ (x + 1)(x + 4) = 0 \]
\[ x^2 + 5x + 4 = \begin{cases} x^2 + 5x + 4, & x \leq -4 \\ -x^2 - 5x - 4, & -4 < x < 1 \end{cases} \]

\(x = -1\), or \(x = -4\)

\(-x^2 - 5x - 4 = 0\)
\(-x^2 - 5x - 4 = 0\)
\(x = 1\), or \(x = -6\)

Solutions are in the domain. Neither solution is in domain. These solutions are extraneous.
Reciprocal Functions

A reciprocal function has the form
\[ y = \frac{1}{f(x)} \]
where \( f(x) \) is a polynomial and \( f(x) \neq 0 \).

For any function \( f(x) \), the reciprocal function is \( \frac{1}{f(x)} \). The reciprocal of \( y = x \) is \( y = \frac{1}{x} \).

To Graph a Reciprocal Function:
- Plot invariant points. Invariant points are where the y values are 1 or -1.
- X-intercepts become vertical asymptotes.
- The x-axis is a horizontal asymptote.
- Take the reciprocal of the y values of the original function to plot the reciprocal of the function.

Restrictions on the denominator of the reciprocal function.

The reciprocal function is not defined when the denominator is 0. These non-permissible values relate to the asymptotes of the graph of the reciprocal function. The non-permissible values for a reciprocal function (position of asymptotes) also come from the x-intercepts of the original graph.

Linear Reciprocal Graphs

Quadratic Reciprocal Graphs

Vocabulary

<table>
<thead>
<tr>
<th>Vocabulary</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute Value</td>
<td>Distance from zero on a number line.</td>
</tr>
<tr>
<td>Piecewise definition</td>
<td>A function that involves the absolute value of a variable.</td>
</tr>
<tr>
<td>Absolute Value Function</td>
<td>A function composed of two or more separate functions or pieces, each with its own specific domain, that combine to define the overall function.</td>
</tr>
<tr>
<td>Invariant Point</td>
<td>A point that remains unchanged when a transformation is applied to it.</td>
</tr>
<tr>
<td>Absolute Value Equation</td>
<td>An equation that includes the absolute value of an expression involving a variable.</td>
</tr>
<tr>
<td>Asymptote</td>
<td>An imaginary line whose distance from a given curve approaches zero.</td>
</tr>
<tr>
<td>Vertical Asymptote</td>
<td>For reciprocal functions, vertical asymptotes occur at the non-permissible values of the function, the x-intercepts of the original function graph.</td>
</tr>
<tr>
<td>Horizontal Asymptote</td>
<td>For our reciprocal functions, there will always be a horizontal asymptote at ( y = 0 ).</td>
</tr>
</tbody>
</table>
### Math 2200 Chapter 8 Systems of Equations Review

<table>
<thead>
<tr>
<th>Key Ideas</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determining the solution of a system of linear-quadratic equations graphically.</td>
<td>Isolate the y variable for each function equation. Graph the line and the parabola on the same grid. The solutions are the points of intersection of the graph, ((x, y)). The ordered pair satisfies both equations. Verify your solutions in the original function equations.</td>
</tr>
<tr>
<td>There are three possibilities for the number of intersection and the number of solutions of a system of linear-quadratic equations.</td>
<td><img src="images" alt="Graphs showing no intersection, one intersection, and two intersections" /></td>
</tr>
<tr>
<td>Determining the solution of a system of quadratic-quadratic equations graphically</td>
<td>Isolate the y variable for each function equation. Graph both parabolas on the same grid. The solutions of a quadratic-quadratic equation are the points of intersection of the two graphs, ((x, y)). Verify the solutions in the original form of the equations.</td>
</tr>
<tr>
<td>There are three possibilities for the number of intersections and the number of solutions of a system of quadratic-quadratic equations. If one quadratic is a multiple of another, there will be an infinite number of solutions.</td>
<td><img src="images" alt="Graphs showing no intersection, one intersection, and two intersections" /></td>
</tr>
<tr>
<td>Determining the solution of a system of linear-quadratic equations algebraically. Two Methods to choose from Substitution or Elimination</td>
<td>Linear Quadratic Systems</td>
</tr>
<tr>
<td><strong>Substitution Method</strong></td>
<td><strong>Elimination Method</strong></td>
</tr>
<tr>
<td>Substitute (x = 15) for (y) in the quadratic equation and simplify. (x^2 + x - 20 = 0) (\Rightarrow (x + 5)(x - 4) = 0) (\Rightarrow x = -5) or (x = 4) For (x = -5) (y = (-5)^2 - 15) (\Rightarrow y = 10) For (x = 4) (y = (4)^2 - 15) (\Rightarrow y = 7) The two solutions are ((-5, 10)) and ((4, 7)).</td>
<td></td>
</tr>
<tr>
<td>Align the terms in the same degree. (y = x^2 + 3x) (-) Since the quadratic term is in the variable (x), eliminate the (y)-term. (0 = x^2 + 2x - 3) (0 = x^2 - 2x - 3) (0 = (x + 3)(x - 1)) Solve the resulting equation. (x = -3) or (x = 1) (x = 1) The two solutions are ((-3, 0)) and ((1, 4)). Subtract to eliminate the (y)-term.</td>
<td></td>
</tr>
</tbody>
</table>
Determine the solution to a system of quadratic-quadratic equations algebraically. You can use Substitution or Elimination.

\[
\begin{align*}
y &= 2x^2 - 2x + 3 \\
y &= x^2 + 5x - 7 \\
2x^2 - 2x + 3 &= x^2 + 5x - 7 \\
x^2 - 7x + 10 &= 0
\end{align*}
\]

The two solutions are (5,43) and (2,7).

Solve the system: \[6x^2 - x - y = -1, \quad 4x^2 - 4x - y = -6\]

\[
\begin{align*}
6x^2 - x - y &= -1 \\
4x^2 - 4x - y &= -6 \\
2x^2 + 3x &= 5 \\
x &= \frac{-5}{2} \\
x &= 1
\end{align*}
\]

The two solutions are (1, 6) and \(\left(\frac{-5}{2}, 41\right)\).

<table>
<thead>
<tr>
<th>Vocabulary</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>System of linear-quadratic equations</td>
<td>A linear equation and a quadratic equation involving the same variables. A graph of the system involves a line and a parabola.</td>
</tr>
<tr>
<td>System of quadratic-quadratic equations</td>
<td>Two quadratic equations involving the same variables. The graph involves two parabolas.</td>
</tr>
<tr>
<td>Solution</td>
<td>With a system of equations or system of inequalities, the solution set is the set containing value(s) of the variable(s) that satisfy all equations and/or inequalities in the system. All the points of intersection of the two graphs. The ordered pairs ((x, y)) that the two function equations have in common.</td>
</tr>
<tr>
<td>Method of substitution</td>
<td>An algebraic method of solving a system of equations. Solve one equation for one variable. Then, substitute that value into the other equation and solve for the other variable.</td>
</tr>
<tr>
<td>Method of elimination</td>
<td>An algebraic method of solving a system of equations. Add or subtract the equations to eliminate one variable and solve for the other variable.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Common Errors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolating a variable</td>
<td>Making errors with signs when isolating a variable.</td>
</tr>
<tr>
<td>Subtraction</td>
<td>Only subtracting the first term when eliminating and adding the other terms.</td>
</tr>
<tr>
<td>Not checking solutions properly</td>
<td>After obtaining your solutions to a quadratic-quadratic or linear-quadratic equation not substituting your solution for (x) and (y) to verify your answer.</td>
</tr>
<tr>
<td>Key Ideas</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>---------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Solving Linear Inequalities in Two Variables</strong></td>
<td>The solution is a shaded half-plane region with a dashed boundary line if the inequality is &lt; or &gt;. The boundary line is solid if the inequality is ≤ or ≥.</td>
</tr>
<tr>
<td>$y \leq -2x + 5$</td>
<td>$y &gt; \frac{1}{3}x + \frac{8}{3}$</td>
</tr>
<tr>
<td>y is “less than or equal to” solid boundary line, shade below</td>
<td>y is “greater than” dashed boundary line, shade above</td>
</tr>
<tr>
<td><strong>Method One:</strong> Isolate the $y$-variable in the linear inequality and graph with technology.</td>
<td><strong>Method Two:</strong> Graph using $x$- and $y$-intercepts and use Test point to determine shaded region. <strong>Method Three:</strong> Isolate the $y$-variable and graph using slope and $y$-intercept, then use test point to determine which side of the boundary line to shade. If the test point makes the inequality “true”, the point lies in the solution region, this side of the boundary should be shaded. If the test point makes the inequality “false” shade the region on the opposite side of the boundary.</td>
</tr>
<tr>
<td><strong>Solving Quadratic Inequalities in One Variable</strong></td>
<td>The solution set contains the intervals of $x$-values where the $y$-values of the graph are above or below the $x$-axis (depending on the inequality). $x^2 + 5x + 6 \geq 0$</td>
</tr>
<tr>
<td><strong>Graphical Method:</strong> Graph the related function; determine zeros and intervals of $x$-values where $y$-values are above or below $x$-axis.</td>
<td><strong>Alternate Method:</strong> Determine the roots of the related function and use Case analysis or sign analysis with test points. Determine zeros of the related function Use test points in each domain region to determine if the function is positive or negative Indicate solution region.</td>
</tr>
<tr>
<td>The solution interval for $(x+2)(x-4) \geq 0$ is ${x \leq -2 \text{ or } x \geq 4}$</td>
<td>The solution interval for $2x^2 + 5x \leq 3$ is written as ${x \leq -3 \text{ or } x \leq \frac{1}{2}}$</td>
</tr>
<tr>
<td><strong>Quadratic Inequalities in Two Variables</strong></td>
<td>The solution is a shaded region with a solid or broken boundary parabolic curve.</td>
</tr>
<tr>
<td>y “is greater than” dashed boundary, shade above</td>
<td>y is “greater than or equal to” solid boundary, shade above</td>
</tr>
</tbody>
</table>